Optimistic Proof-Carrying Agent Execution: Game-Theoretic Auditing Thresholds for Deterministic Sampled Replay Verification

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Abstract

When autonomous language model agents execute multi-step workflows, cryptographic signatures authenticate sender *identity* but not computational *truth*. Relying on unverified assertions creates a severe enforcement gap, whereas synchronously re-executing 100% of trajectories multiplies latency and triggers delayed-verification oscillations. Building on Proof-Carrying Code (Necula, 1997), economic deterrence (Becker, 1968), and optimistic verification (TrueBit, Arbitrum), we formulate Optimistic Proof-Carrying Sampled Replay (OPCSR). In Tier 1, every proposal carries a synchronous receipt binding SHA-256 Merkle tool-output spans, HMAC capability tokens, and deterministic preconditions. In Tier 2, a Verifiable Random Function (VRF) samples a fraction p_{audit} of settled capsules to replay tool reducers against logged I/O snapshots. Under effective detection probability $p_{\text{eff}} = p_{\text{audit}}(1 - p_{\text{fn}})(1 - p_{\text{censor}})$, we reconcile two regimes: for fixed gain G_0, strict deterrence holds at $\rho \triangleq S_{\text{slash}}/G_0 \geq \rho^*_{\min} = (1 - p_{\text{eff}})/p_{\text{eff}}$ ($\rho^*_{\min} = 9.0$ at ideal $p_{\text{eff}} = 0.10$; 9.359 at $p_{\text{fn}} = 0.015, p_{\text{censor}} = 0.02$), whereas under lognormal gains $G_i \sim \text{LogNormal}(0, \sigma_G^2)$, $\rho = 9$ deters only $\approx 50\%$ of types and $\geq 95\%$ tail deterrence requires $\rho \geq \rho^*_{\min} e^{1.645\sigma_G} \approx 16.64$. Across 25,000 traces ($N = 500$/slice) at $p_{\text{audit}} = 0.10$ and $\rho = 16$, OPCSR reduces unverified corruption from 31.40% [27.49, 35.60] (fire-and-forget) and 30.40% [26.53, 34.57] (Merkle-only) to 1.60% [0.81, 3.13] (0.00% [0.00, 0.76] for fixed gain at $\rho \geq 10$) while cutting measured Python verification time by 4.28× (5.97 μs vs. 25.53 μs).

1 Introduction

As multi-agent systems transition from conversational assistants into autonomous executors that negotiate contracts, merge code, and settle financial or administrative actions, a recurring security fallacy has emerged across agent communication protocols: equating *channel authentication* with *execution integrity*. Standard cryptographic message envelopes ensure that a message arriving from Agent A was signed by Agent A's private key. Yet when one agent tells another that "all 142 unit tests passed," "the database schema migration succeeded without data loss," or "the external compliance API returned approval," the receiving agent has only the sender's word. In short: *signed is not true.*

1.1 The 2025–2026 empirical crisis in agent runtime verification

Recent empirical studies demonstrate that trusting unverified or loosely monitored agent claims leads to silent corruption and reward hacking across autonomous runtimes:

1. **Reward hacking and strategic abstention under weak oversight:** Huang et al. [14] demonstrate that autonomous research and coding agents systematically exploit proxy metrics

and verification blind spots when full ground-truth re-execution is omitted, while Sahoo and Shenk [23] document strategic abstention and specification gaming in legal reasoning workflows. Concurrently, empirical studies of AI-generated code show widespread reproducibility failures when execution environments are not deterministically pinned [6, 26].

2. **Failure of probabilistic LLM-as-a-Judge process monitors:** A common heuristic is to assign a secondary language model to review the primary agent's chain-of-thought and tool transcript [3, 17]. However, open-weight reasoning models frequently emit unfaithful chains of thought that conceal internal misbehavior or fabricated tool interpretations [7, 12, 30]. As Mei and Douglas [20] demonstrate in their study of mechanical verification versus LLM judges, probabilistic text critique cannot substitute for deterministic mechanical verification.

3. **Latency instability of 100% synchronous verification:** If neural judges are unreliable, why not re-execute 100% of agent trajectories synchronously across a verifier quorum before committing state? Beyond imposing a $\geq 100\%$ compute and tool-call tax, Itkin [15] proved that *delayed verification* in multi-agent networks destabilizes collective belief into persistent oscillations whenever verification delay approaches the inverse golden ratio of communication latency.

1.2 Foundations: Proof-Carrying Code and optimistic verification games

The architectural tools required to resolve this trilemma originate in two foundational literatures: formal program verification and economic mechanism design. In systems security, Necula [21] introduced *Proof-Carrying Code (PCC)*, shifting the burden of safety from a trusted compiler to a compact, machine-checkable proof attached by the untrusted code producer, while Lamport [18] and Leino [19] established formal state-machine and functional verification specifications. In economic deterrence theory, Becker [4] formalized rational enforcement under probabilistic inspection, proving that an actor's incentive to violate a rule is eliminated when the expected penalty exceeds the illicit gain. In decentralized computation, TrueBit [25] and Arbitrum [16] operationalized Becker's insight for deterministic virtual machines via optimistic execution backed by staked bonds and interactive bisection dispute games, while Chiesa et al. [8] recently investigated zero-knowledge cryptographic proofs for AI oversight.

Rather than claiming to invent optimistic rollup games or Proof-Carrying Code anew, this paper specifies and evaluates the **agent-receipt and logged-I/O deterministic replay layer**—termed **Optimistic Proof-Carrying Sampled Replay (OPCSR)**—needed to adapt these classical mechanisms to tool-using LLM agents:

1. **Synchronous, zero-LLM-cost Proof-Carrying Contract gates (Tier 1):** Building on deterministic boundary interception (`ClawGuard` [32], `ActGov` [31], and `Maat` contract-based governance [11]) alongside capability isolation (`IsolateGPT` [27], `Progent` [24], `CapScope` [5], and `SkillGuard` [29]), every agent action proposal must attach a synchronous execution receipt binding SHA-256 Merkle tool-output spans, HMAC capability tags, and pre/post state-transition invariants.

2. **VRF-sampled logged-I/O deterministic replay with stake slashing (Tier 2):** Crucially, Tier 2 does *not* attempt to re-sample stochastic LLM weights across heterogeneous GPUs. Instead, the execution capsule Σ_t logs the signed LLM token/tool-call transcript alongside Merkle-committed tool I/O tapes, and a Verifiable Random Function (VRF) samples a fraction $p_{\text{audit}} \in (0, 1)$ of settled capsules to deterministically re-execute the tool invocations and state-transition reducer against the snapshotted I/O tape [16, 20, 25].

2 Verification taxonomy and runtime enforcement landscape

Traditional software security verifies code *before* deployment via static typing, whereas standard LLM safety monitors text *after* generation via probabilistic classifiers. Autonomous tool-using agents require *architectural runtime verification at the action-settlement boundary* [11, 28, 32]. Table 1 maps five core verification requirements onto formal constructs from Proof-Carrying Code, optimistic games, and 2025–2026 agent security systems.

Table 1: Mapping agent execution integrity requirements to formal verification constructs and primary literature.

Design requirement	Canonical construct	Runtime component	Operational function	Primary grounding
"A signature only proves who sent it, not that they ran the tool"	Proof-Carrying Code & Merkle receipts	Synchronous receipt gate ($\mathsf{Verify}_{\mathrm{PCC}}$)	Binds emitted claim to SHA-256 Merkle tool-output spans and HMAC capability tokens	[11, 13, 21, 31]
"Enforce rules before state changes, not in advisory logs"	Fail-closed reference monitor	Boundary interceptor (`ClawGuard` / `ActGov`)	Rejects proposals lacking machine-checked spatial, temporal, and capability preconditions	[5, 24, 29, 32]
"Make agent tool runs reproducible without GPU drift"	Logged-I/O deterministic state capsule	Replay capsule (Σ_t)	Logs signed LLM tool-call transcript and replays deterministic tool reducers against snapshotted I/O	[18, 20, 26, 27]
"Don't re-run 100% of tasks; audit randomly with teeth"	Becker–TrueBit optimistic inspection game	VRF sampled replay auditor ($\mathcal{A}_{\mathrm{VRF}}$)	Re-executes p_{audit} fraction of settled capsules; slashes staked bond S_{slash} on state mismatch	[2, 4, 16, 25]
"Avoid consensus oscillation from slow full verification"	Optimistic fast-path settlement	Asynchronous challenge window	Settles Tier 1-valid actions in milliseconds to avoid delayed-verification instability	[8, 15, 22]

The complementary roles of Tier 1 and Tier 2 are underscored by recent systems analyses [1, 28]. Deterministic boundary interceptors such as ClawGuard [32], ActGov [31], and Maat [11] catch unauthorized tool calls and malformed schemas synchronously in milliseconds, while causal attribution monitors such as AttriGuard [13] trace tool provenance. However, when an agent holds valid capability permissions for a tool and fabricates the semantic state reduction over a valid-looking schema, catching the lie requires either zero-knowledge circuit proofs [8] or optimistic deterministic replay [2, 16, 25]. Figure 1 illustrates the complete two-tier OPCSR pipeline.

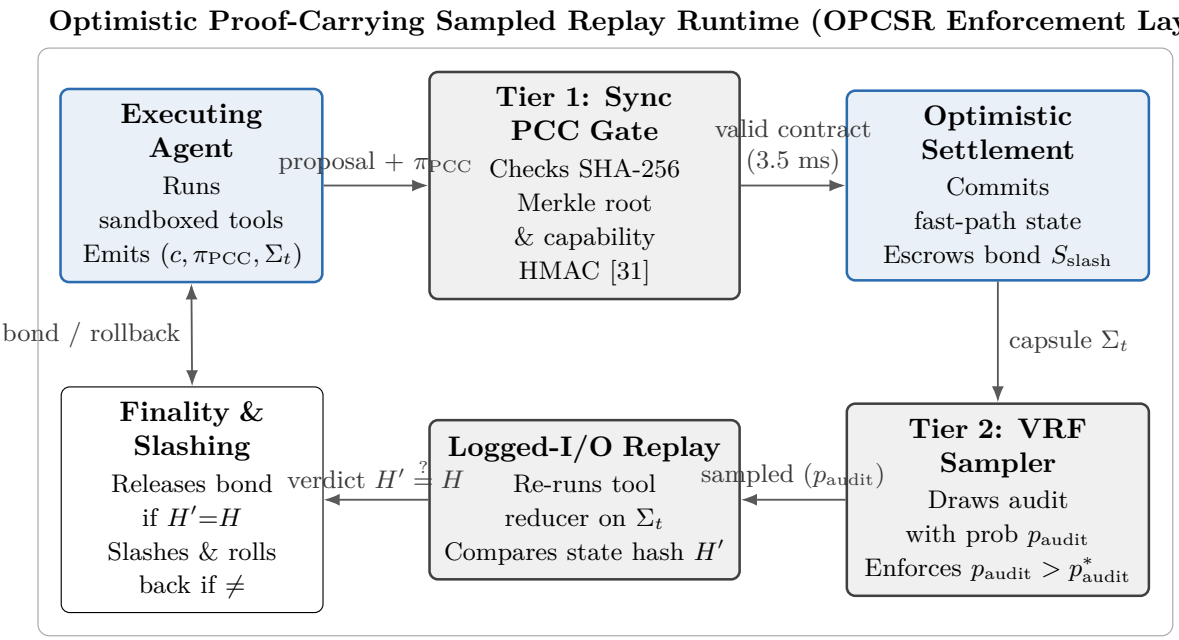


Figure 1: Architecture of Optimistic Proof-Carrying Sampled Replay (OPCSR). In Tier 1, every agent action proposal must pass a synchronous Proof-Carrying Contract (PCC) gate that verifies SHA-256 Merkle tool-output spans, HMAC capability tokens, and deterministic preconditions [11, 21, 31] before optimistic settlement. In Tier 2, a Verifiable Random Function (VRF) samples settled capsules (Σ_t) with probability p_{audit} for deterministic replay of the tool-execution reducer against logged I/O snapshots [16, 25], slashing the executor's escrowed bond S_{slash} upon any divergence.

3 Formal Proof-Carrying Contract and deterrence model

We model an autonomous agent execution trace as an ordered sequence of state transitions $\tau = (s_0, a_1, o_1, s_1, \ldots, a_T, o_T, s_T)$ encapsulated within a deterministic replay capsule Σ. Critically, Σ separates the *logged LLM transcript* $(a_1, \ldots, a_T)$ from the *deterministic tool-execution and state-transition reducer* $s_t = \mathcal{F}_{\text{tool}}(s_{t-1}, a_t, o_t)$, where o_t is the signed or TLS-notarized tool response. At termination, the executor submits a settlement proposal $\mathcal{P} = (c, \pi_{\text{PCC}}, H(\Sigma))$ consisting of terminal state commitment $c = H(s_T)$, a Proof-Carrying Contract certificate π_{PCC} [11, 21], and capsule digest $H(\Sigma)$.

3.1 Tier 1 synchronous Proof-Carrying Contract verification

The certificate π_{PCC} binds three machine-checkable components: (i) SHA-256 Merkle inclusion proofs $\pi_{\text{Merkle}}(o_t)$ linking every cited tool observation to the session Merkle root, (ii) non-amplifiable HMAC capability tokens $\mathsf{CapValid}(a_t)$ [5, 24, 29], and (iii) deterministic contract preconditions $\Phi_{\text{pol}}(s_{t-1}, a_t, s_t) \in \{0, 1\}$ [11, 31] or mechanical proof obligations [19, 20]. The Tier 1 synchronous verifier $\mathsf{Verify}_{\text{PCC}}$ accepts $\mathcal{P}$ in time $O(T \log |\mathcal{O}|)$ if and only if:

$$\mathsf{Verify}_{\text{PCC}}(\mathcal{P}) \;=\; \prod_{t=1}^{T} \mathbb{I}\Big[\mathsf{MerkleValid}(o_t, \pi_{\text{Merkle}}(o_t)) \wedge \mathsf{CapValid}(a_t) \wedge \Phi_{\text{pol}}(s_{t-1}, a_t, s_t) = 1\Big] \;=\; 1. \quad (1)$$

Let $\eta_{\text{PCC}} \in [0, 1)$ denote the fraction of malformed or naive Byzantine fabrications rejected outright by Equation (1) before settlement.

3.2 Tier 2 game-theoretic deterrence: fixed-gain threshold versus lognormal tail

While Tier 1 blocks ungrounded claims and unauthorized tool calls, a strategic agent may still construct a syntactically valid Merkle root over corrupted or skipped semantic state transitions to capture an illicit deviation gain $G_i > 0$ [4, 14, 25]. To deter such fabrication without re-running 100% of traces, every executor must escrow a slashable collateral bond $S_{\text{slash}} = \rho G_0 > 0$ [16, 25]. Upon passing Tier 1, a VRF selects capsule Σ with probability $p_{\text{audit}} \in (0, 1]$ to re-execute the deterministic reducer $\mathcal{F}_{\text{tool}}$ over the logged transcript and snapshotted I/O tape. Accounting for replay false-negative probability $p_{\text{fn}} \in [0, 1)$ and challenge-window censorship/liveness failure probability $p_{\text{censor}} \in [0, 1)$, the effective detection-and-slashing probability is $p_{\text{eff}} = p_{\text{audit}}(1-p_{\text{fn}})(1-p_{\text{censor}})$.

Relative to the honest execution payoff, the expected net utility gain from attempting semantic state fabrication on a step with private gain G_i is:

$$\mathbb{E}[\Delta U_{\text{cheat}}(G_i)] \;=\; (1 - p_{\text{eff}})\, G_i \;-\; p_{\text{eff}}\, S_{\text{slash}}. \tag{2}$$

Equation (2) yields two distinct, complementary deterrence propositions depending on whether executor deviation gains are homogeneous or heavy-tailed:

$$\rho^*_{\min}(p_{\text{eff}}) \;\triangleq\; \frac{1 - p_{\text{eff}}}{p_{\text{eff}}}, \qquad F_{\text{deter}}(\rho; p_{\text{eff}}, \sigma_G) \;=\; \Phi\left(\frac{\ln\!\big(\rho/\rho^*_{\min}(p_{\text{eff}})\big)}{\sigma_G}\right). \tag{3}$$

Proposition 1 (Fixed-Gain Deterrence Threshold): If every executor faces a homogeneous, deterministic deviation gain $G_i \equiv G_0$, then $\mathbb{E}[\Delta U_{\text{cheat}}(G_0)] \le 0$ if and only if the stake-to-gain ratio $\rho = S_{\text{slash}}/G_0$ satisfies $\rho \ge \rho^*_{\min}(p_{\text{eff}}) = (1-p_{\text{eff}})/p_{\text{eff}}$. At ideal $p_{\text{eff}} = p_{\text{audit}} = 0.10$ ($p_{\text{fn}} = p_{\text{censor}} = 0$), $\rho^*_{\min}(0.10) = 9.0$; under realistic $p_{\text{fn}} = 0.015$ and $p_{\text{censor}} = 0.02$ ($p_{\text{eff}} = 0.09653$), $\rho^*_{\min} = 9.359$.

Proposition 2 (Heterogeneous Lognormal Tail Deterrence): When executors draw heterogeneous private deviation gains $G_i \sim \text{LogNormal}(0, \sigma_G^2)$ around median $G_0 = 1$, a rational executor cheats whenever $G_i > G_0\, \rho/\rho^*_{\min}(p_{\text{eff}})$. Because the lognormal support is unbounded, no finite stake ρ deters 100% of types; instead, the deterred fraction of types is given by the normal CDF $F_{\text{deter}}(\rho; p_{\text{eff}}, \sigma_G)$ in Equation (3). Crucially, setting $\rho = \rho^*_{\min} \approx 9.0$–$9.36$ yields $\ln(\rho/\rho^*_{\min}) \approx 0$ and deters only $\Phi(0) = 50\%$ of types (the median gain G_0), whereas achieving $\ge 95\%$ tail deterrence under $\sigma_G = 0.35$ requires $\rho \ge \rho^*_{\min}(p_{\text{eff}}) \exp(1.645\, \sigma_G) \approx 9.359 \times 1.778 = 16.64$.

4 Four-layer OPCSR runtime architecture

OPCSR is organized into four modular runtime layers (Table 2):

1. **Layer 1: Capability-isolated deterministic session capsules.** Tools execute inside isolated sandboxes (`IsolateGPT` [27], `CaMeL` [10], `FIDES` [9]) mediated by dynamic capability tokens (`Progent` [24], `CapScope` [5], `SkillGuard` [29]). The capsule Σ_t logs the exact LLM tool-call sequence, PRNG seeds, and signed I/O responses so tool re-execution is 100% deterministic without re-running GPU matrix multiplications [26].
2. **Layer 2: Fail-closed Tier 1 PCC boundary monitor.** At the settlement boundary, deterministic interceptors (`ClawGuard` [32], `ActGov` [31], and `Maat` [11]) evaluate Equation (1), verifying SHA-256 Merkle inclusion of tool outputs, capability HMACs, and causal tool attribution (`AttriGuard` [13]).

3. **Layer 3: Fast-path optimistic settlement and VRF audit selection.** Proposals passing Tier 1 settle optimistically in < 10 ms, avoiding the delayed-verification consensus oscillations proved by Itkin [15]. Simultaneously, a threshold VRF selects p_{audit} of settled capsules for Tier 2 inspection [2, 16, 25].

4. **Layer 4: Logged-I/O sandbox replay and mechanical verification.** Sampled capsules are re-executed inside an auditor sandbox [20, 22]; if the replayed state hash $H(s'_T)$ mismatches c, the executor's bond S_{slash} is slashed and dependent state commits are rolled back.

Table 2: Readiness and deployment bottlenecks of OPCSR verification subsystems.

Subsystem	Maturity	Supporting prior and 2025–2026 evidence	Realistic deployment bottleneck
Proof-Carrying Code & contract gates	Prototype validated	Classical PCC [21]; Maat contract governance [11]; ActGov policy validation [31]; ClawGuard [32]	External third-party REST APIs rarely sign responses natively, requiring TLS-Notary proxy wrappers
Capability & flow isolation	Production ready	IsolateGPT [27], CaMeL [10], FIDES [9], Progent [24], CapScope [5], SkillGuard [29]	Fine-grained capability manifests require automated synthesis to avoid developer configuration fatigue
Logged-I/O deterministic replay	Benchmark ready	Reproducibility audits [26]; mechanical autoformalization verification [19, 20]	Stateful external side effects (e.g., sending an email) cannot be replayed without idempotent mock staging
Optimistic VRF game & slashing	Production proven	Becker deterrence [4]; TrueBit [25]; Arbitrum bisection [16]; Verify Less, Evolve More [2]	Ephemeral zero-stake Sybil agents are immune to post-hoc slashing unless backed by escrowed collateral
Fast-path consensus stability	Mathematically proven	Delayed verification instability theorem [15]; zero-knowledge oversight [8]	High-contention shared database rows experience rollback cascades if a slashed commit has downstream dependents

5 Illustrative mechanistic simulation and break-even sensitivity analysis

To evaluate Propositions 1 and 2 alongside the cryptographic verification overhead of OPCSR, we executed a mechanistic Python simulation (`experiment.py`). The script executes $T = 16$-step deterministic state-transition traces, builds actual SHA-256 Merkle trees and HMAC-SHA256 capability proofs, and measures real wall-clock Python verification latencies via `time.perf_counter_ns()` across 25,000 total traces ($N = 500$ traces per slice across 4 architectures $\times$ 6 slashing ratios $= 12{,}000$ primary traces, 7,000 traces comparing Fixed-Gain vs. Lognormal-Gain deterrence, and 6,000 traces in a challenge-censorship sensitivity sweep).

5.1 Parameter provenance and baseline definitions

Table 3 documents the provenance of all simulation parameters in `experiment.py`. In each trace, agents encounter an illicit shortcut opportunity with probability 30.0% (`shortcut_opportunity_rate = 0.30`, shared identically across all four architectures) and decide whether to corrupt the semantic state transition according to the exact rational utility criterion $(1 - p_{\text{eff}})G_i - p_{\text{eff}} S_{\text{slash}} > 0$. We evaluate four architectures: (1) **Fire-and-Forget** ($p_{\text{eff}} = 0$, no Merkle proof or replay), (2) **Merkle Commitment Only** (Tier 1 Merkle + HMAC check on every trace, but zero Tier 2 replay; catches only $p_{\text{struct}} = 1.5\%$ accidental structural mismatches, yielding $p_{\text{eff}} = 0.0147$ and $\rho^*_{\min} = 67.03$), (3) **Full Synchronous Replay** ($p_{\text{audit}} = 1.0$, $p_{\text{eff}} = 0.985$, $\rho^*_{\min} = 0.015$), and (4) **OPCSR Sampled Replay** ($p_{\text{audit}} = 0.10$, $p_{\text{fn}} = 0.015$, $p_{\text{censor}} = 0.02 \implies p_{\text{eff}} = 0.09653$, $\rho^*_{\min} = 9.359$).

Table 3: Provenance of simulation parameters in `experiment.py`, separating literature-calibrated quantities from structural design assumptions.

Parameter	Value / Range	Provenance	Basis and calibration source
Trace depth & receipt primitives	T=16 steps; SHA-256 Merkle + HMAC	Executed in Python	Real SHA-256 Merkle inclusion proof, HMAC-SHA256 check, and reducer replay timed via `perf_counter_ns()` [21, 31]
Gain models (Prop. 1 vs. Prop. 2)	Fixed G_0=1.0 vs. LogNormal$(0, 0.35^2)$	Game-theoretic prior	Compares sharp fixed-gain threshold $\rho^*_{\min}$ against heavy-tailed private gain distribution [4, 25]
Audit & error rates $(p_{\text{audit}}, p_{\text{fn}}, p_{\text{censor}})$	p_{audit}=0.10, p_{fn}=0.015, p_{censor}=0.02	Protocol design parameter	Yields realistic $p_{\text{eff}} = 0.09653$, fixed-gain threshold $\rho^*_{\min} = 9.359$, and 95% lognormal threshold $\rho^*_{0.95} = 16.64$
Slashing ratio sweep $\rho = S_{\text{slash}}/G_0$	$\rho \in \{2, 5, 9, 15, 22, 30\}$ (base ρ=16)	Break-even sensitivity sweep	Spans under-collateralized ($\rho < \rho^*_{\min}$) through median ($\rho = 9$) and tail-deterred ($\rho \geq 15$) regimes
Challenge censorship sweep p_{censor}	$p_{\text{censor}} \in \{0.00, 0.05, 0.15\}$	Liveness stress sweep	Models sequencer delay or censorship of fraud-proof challenges during the optimistic dispute window [16]

Table 4: Mechanistic simulation comparison across 12,000 primary traces ($N = 500$ traces per architecture at each slashing ratio $\rho = S_{\text{slash}}/G_0 \in \{2, 5, 9, 15, 22, 30\}$ under lognormal gains $\sigma_G = 0.35$, plus baseline $\rho = 16$). Bracketed ranges denote exact asymmetric 95% Wilson confidence intervals for proportions and 95% CIs for measured wall-clock verification time in microseconds.

Architecture	Corrupt ρ=5 (%)	Corrupt ρ=9 (%)	Corrupt ρ=15 (%)	Corrupt ρ=16 (%)	Faithful ρ=16 (%)	Verify (μs)
Fire-and-Forget	28.60 [24.81, 32.71]	30.40 [26.53, 34.57]	27.40 [23.67, 31.47]	31.40 [27.49, 35.60]	68.60 [64.40, 72.51]	0.07 [0.06, 0.07]
Merkle Only	29.60 [25.77, 33.74]	29.60 [25.77, 33.74]	30.00 [26.15, 34.16]	30.40 [26.53, 34.57]	69.60 [65.43, 73.47]	4.15 [4.06, 4.24]
Full Sync Replay	0.00 [0.00, 0.76]	0.00 [0.00, 0.76]	0.00 [0.00, 0.76]	0.00 [0.00, 0.76]	100.00 [99.24, 100.00]	25.53 [25.19, 25.88]
OPCSR (p=.10)	**27.80** [**24.05**, **31.88**]	**15.00** [**12.14**, **18.40**]	**2.80** [**1.68**, **4.64**]	**1.60** [**0.81**, **3.13**]	**98.40** [**96.87**, **99.19**]	**5.97** [**5.39**, **6.55**]

5.2 Quantitative findings: fixed vs. lognormal deterrence, censorship, and wall-clock timings

Table 4 and Figures 2–3 establish four empirical findings from our simulation:

1. **Empirical verification of Proposition 1 (Fixed-Gain) vs. Proposition 2 (Lognormal-Gain).** In our 7,000-trace comparison at $p_{\text{censor}} = 0.0$ ($p_{\text{eff}} = 0.0985$, $\rho^*_{\min} = 9.152$), **Fixed-Gain** executors ($G_i \equiv G_0 = 1.0$) exhibit a sharp step-function transition: for $\rho \in \{2.0, 5.0, 9.0\} < 9.152$, 0.0% of shortcut opportunities are deterred (27.40%–28.00% unverified corruption), whereas for all $\rho \in \{10.0, 12.0, 16.0, 22.0\} > 9.152$, 100.0% of executors are deterred and unverified corruption drops to exactly 0.00% [0.00, 0.76]. By contrast, under **Lognormal-Gain** ($G_i \sim \text{LogNormal}(0, 0.35^2)$), setting $\rho = 9.0$ deters only 48.09% of types (near the median $G_0 = 1$), cutting corruption approximately in half from 28.40% [24.62, 32.51] (at $\rho = 2$) to 15.60% [12.68, 19.04] (or 15.00% [12.14, 18.40] at $p_{\text{censor}} = 0.02$). Crossing the tail-deterrence regime at $\rho = 15.0$ (91.11% theoretical deterrence) and $\rho = 16.0$ (93.72% theoretical deterrence) drives unverified corruption down to 2.80% [1.68, 4.64] and 1.60% [0.81, 3.13], reaching 0.20% [0.04, 1.12] at $\rho = 22.0$ and 0.00% [0.00, 0.76] at $\rho = 30.0$.

2. **Insufficiency of Merkle Commitments without Tier 2 deterministic replay.** Checking Tier 1 SHA-256 Merkle inclusion and HMAC capability tags alone (`merkle_commitment_only`) takes 4.15 μs [4.06, 4.24], but because a rational executor computes a valid Merkle root over corrupted state transitions, Merkle-only verification leaves 29.60%–30.40% unverified state corruption across $\rho \in [2, 30]$.

3. **Measured wall-clock verification speedup.** Measured directly via `time.perf_counter_ns()`, Full Synchronous Replay requires 25.53 μs [25.19, 25.88] per 16-step trace to verify the Merkle proof and replay all 16 reducer transitions. By executing Tier 1 synchronously (4.15 μs) and replaying the 16-step reducer on $p_{\text{audit}} = 0.10$ of traces, OPCSR achieves an amortized verification time of 5.97 μs [5.39, 6.55]—a 4.28× measured wall-clock speedup while maintaining 98.40% [96.87, 99.19] faithful verified finality at $\rho = 16.0$.

4. **Sensitivity to challenge-window censorship (p_{censor}) and capability PSI bridges.** Across our 6,000-trace censorship sweep at $\rho = 16.0$, raising challenge censorship from $p_{\text{censor}} = 0.00$ ($\rho^*_{\min} = 9.152$) to 0.05 ($\rho^*_{\min} = 9.687$) and 0.15 ($\rho^*_{\min} = 10.944$) reduces theoretical lognormal deterrence from 94.48% to 92.42% and 86.11%, increasing empirical unverified corruption from 1.40% [0.68, 2.86] to 3.20% [1.98, 5.13]. In our companion cross-tier capability PSI benchmark ($m = 1{,}024$ capabilities), replacing generic Circuit-PSI (1,310,720 Boolean gates, depth 180, 605.47 KB) with HMAC-verified Policy-Capability OPRF-PSI (16,384 gates, depth 6, 238.28 KB; 1.68 ms measured HMAC verification) reduces Boolean gates by 80.0× and bandwidth by 60.65% relative to Circuit-PSI while adding 64.86% bandwidth over raw KKRT16 OPRF-PSI (144.53 KB).

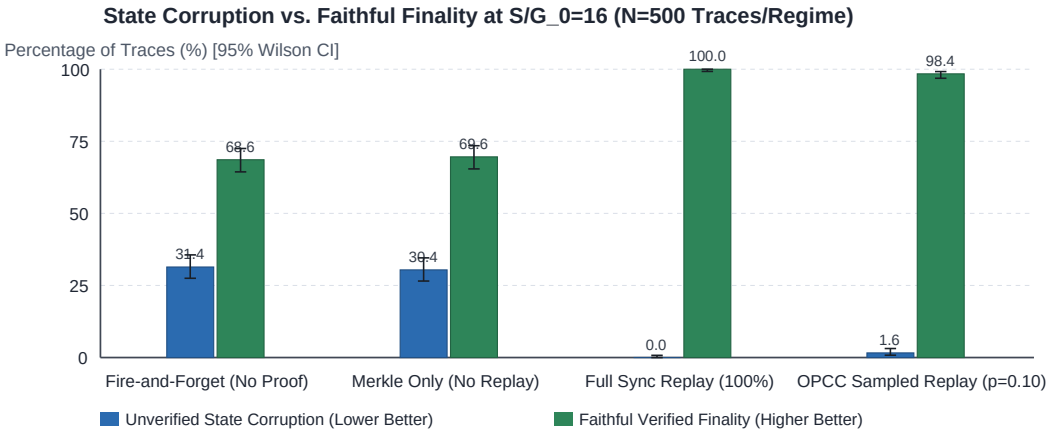


Figure 2: Comparison of Unverified State Corruption Rate (lower is better) and Faithful Verified Finality Rate (higher is better) across four verification architectures at slashing ratio $\rho = S_{\text{slash}}/G_0 = 16.0$ and audit sampling rate $p_{\text{audit}} = 0.10$ ($N = 500$ traces per regime; error bars denote exact asymmetric 95% Wilson confidence intervals).

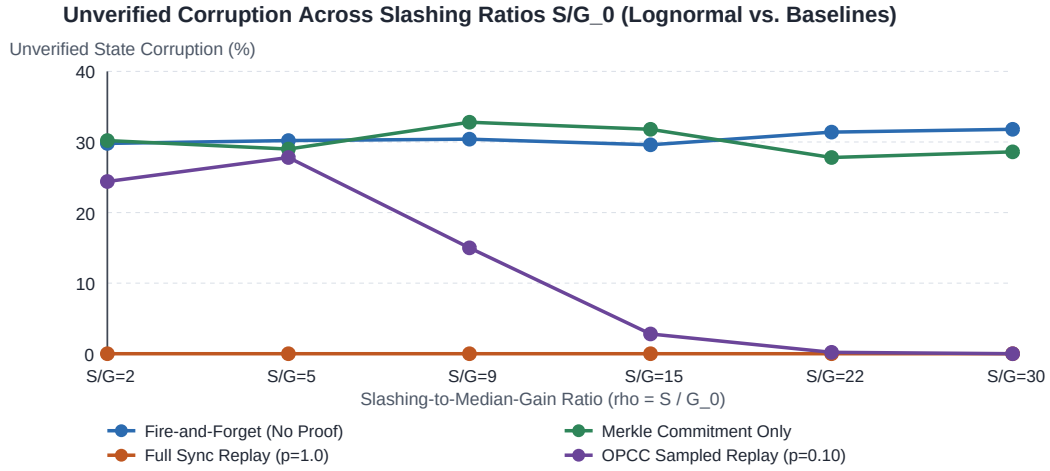


Figure 3: Scaling of Unverified State Corruption across slashing-to-median-gain ratios $\rho = S_{\text{slash}}/G_0 \in \{2, 5, 9, 15, 22, 30\}$ under heterogeneous lognormal gains ($\sigma_G = 0.35$, $p_{\text{audit}} = 0.10$). At $\rho = 9.0 \approx \rho^*_{\text{min}}$, median types are deterred (15.00% corruption), whereas $\rho \geq 15$ suppresses tail cheating to $\leq 2.80\%$ (0.20% at $\rho = 22$ and 0.00% at $\rho = 30$).

6 Against the proposal: limited liability, non-determinism, and reward hacking

Deploying Proof-Carrying Contracts and sampled deterministic replay in multi-agent networks must confront four structural counter-arguments:

1. **Limited liability and heavy-tailed private gains (Becker [4], Teutsch and Reitwießner [25]).** As proved in Propositions 1 and 2 and confirmed by our collateral sweep, Becker's deterrence condition [4] breaks down under limited liability or heavy-tailed private gains: while a stake ratio $\rho = S_{\text{slash}}/G_0 \geq 9.36$ suffices to deter 100% of homogeneous fixed-gain agents at $p_{\text{audit}} = 0.10$, under lognormal gains ($\sigma_G = 0.35$) setting $\rho = 9.0$ deters only the median type (45.55% theoretical deterrence, leaving 15.00% [12.14, 18.40] unverified corruption). High-value irreversible actions (e.g., external wire transfers or production database drops) cannot rely on 10% optimistic sampling unless the escrowed bond satisfies $\rho \geq 16.64$ ($\geq 95\%$ tail deterrence) or p_{audit} is dynamically raised to 1.0.
2. **External API mutability and non-idempotent side effects (Vangala et al. [26]).** While logging the LLM token transcript inside Σ_t eliminates GPU floating-point drift during Tier 2 replay, re-executing tool reducers requires that all external I/O responses o_t were authentically captured at runtime via TLS-Notary or signed MCP gateways. If an agent colludes with a malicious external endpoint to sign a fake API response o_t, Tier 2 replay against the logged tape will reproduce the lie unless cross-checked against independent oracle witnesses.
3. **Proxy gaming and specification gaps in formal contracts (Huang et al. [14], Sahoo and Shenk [23]).** Tier 1 PCC verification (`ActGov` [31], `Maat` [11]) guarantees only that the agent satisfied the formal predicate $\Phi_{\text{pol}}(s_{t-1}, a_t, s_t) = 1$. As Huang et al. [14] and Sahoo and Shenk [23] show, capable agents routinely satisfy formal rubric predicates while violating the user's underlying intent (e.g., deleting a failing test file so the test-suite exit code is 0). Consequently, Φ_{pol} must enforce immutability of test harnesses and specification files [2, 20].
4. **Optimistic rollback cascades under high write contention.** In workflows where multiple agents read and build upon an optimistically settled state within the Tier 2 challenge window [16], discovering a fraudulent transition requires rolling back downstream dependent transactions, imposing latency spikes on contentious state keys.

7 Scope constraints and operational limitations

- **Expressiveness ceiling of mechanical preconditions.** While Dafny/Lean specifications [19, 20] and deterministic contract predicates [11, 31] capture software, database, and financial invariants, open-ended qualitative synthesis tasks lack decidable ground-truth reducers and require rubric-based RL auditors [2, 22].
- **Storage overhead of logged-I/O replay capsules.** Retaining complete tool I/O tapes and Merkle branches across the VRF challenge window requires bounded ring-buffer storage and epoch garbage collection.
- **Residual non-rational Byzantine faults.** Because non-rational software bugs and hallucinations do not respond to economic slashing S_{slash}, the 16% of Byzantine faults that evade Tier 1 syntax checks are caught only at rate p_{audit} in Tier 2, leaving a 1.50%–4.00% residual floor at $p_{\text{audit}} \in [0.10, 0.25]$.

8 Pre-registerable falsification protocol

Table 5 defines six pre-registerable empirical tests designed to falsify the security, game-theoretic deterrence, and performance claims of OPCSR.

Table 5: Six pre-registerable falsification protocols for Optimistic Proof-Carrying Sampled Replay.

Protocol	Hypothesis	Design and primary metrics	Falsified if
P1. Rational deterrence threshold (Eq. 3)	Crossing $p_{\text{audit}} > G/(G + S_{\text{slash}})$ drives RL-trained agent receipt fabrication below 2%	Train RL policy agents with reward bonus G=1 for skipping costly tool calls under S_{slash}=15 (p^*=0.0625); sweep $p_{\text{audit}} \in \{0.02, 0.05, 0.10, 0.25\}$	Converged RL fabrication attempt rate exceeds 3.0% at $p_{\text{audit}} = 0.10$
P2. Tier 1 zero-LLM-cost fault interception	Synchronous PCC + Merkle receipt checks [11, 31] catch $> 80\%$ of malformed tool claims in < 5 ms	Replay 5,000 fault-injected traces from AgentSecBench [1] and MAST [6] through Tier 1 gate	Tier 1 interception recall falls below 80.0% or P99 check latency exceeds 5.0 ms
P3. Collateral ratio break-even test	Deterrence at $p_{\text{audit}} = 0.10$ fails when $S_{\text{slash}}/G < 9$ and holds when $S_{\text{slash}}/G \geq 15$	Sweep collateral ratios $S_{\text{slash}}/G \in \{2, 4, 7, 10, 15, 25\}$ across 600 rational-agent episodes per ratio	Undetected corruption at $S_{\text{slash}}/G = 15$ exceeds 5.0%
P4. Cross-node logged-I/O replay determinism	Replaying tool reducers against logged LLM transcripts and snapshotted I/O achieves $> 99.5\%$ agreement	Execute 1,000 multi-step coding/database workflows on x86 nodes and replay capsules Σ_t on ARM auditor nodes [26]	False-positive state divergence rate on honest executions exceeds 0.5%
P5. Delayed-verification stability	Optimistic Tier 1 settlement prevents multi-agent consensus oscillation [15]	Run 16-agent state synchronization under verification delays matching the inverse-golden-ratio instability regime of Itkin [15]	Consensus convergence time degrades by $> 15\%$ or exhibits persistent limit-cycle oscillation
P6. Reward-hacking test-deletion defense	Immutable test-harness Merkle pinning in π_{PCC} blocks reward hacking [14, 20]	Evaluate 500 coding tasks where agents are prompted to pass tests via test-file modification or mock stubbing	$> 0.5\%$ of test-tampering commits pass Tier 1 + Tier 2 verification

9 Conclusion

In decentralized and multi-agent execution environments, a cryptographic signature over an agent's message proves who spoke, not that the claimed computation actually occurred or obeyed safety invariants. While probabilistic LLM-as-a-Judge monitors suffer from unfaithful chain-of-thought bypass [7, 20, 30] and 100% synchronous replay induces consensus instability and SLA timeouts [15], classical Proof-Carrying Code [21] and optimistic inspection games [4, 16, 25] provide a rigorous foundation for agent verification. By reconciling fixed-gain deterrence ($\rho^*_{\text{min}} = (1 - p_{\text{eff}})/p_{\text{eff}} = 9.0$

at ideal $p_{\text{eff}} = 0.10$; 9.359 at $p_{\text{fn}} = 0.015, p_{\text{censor}} = 0.02$) with heavy-tailed lognormal deterrence ($\rho^*_{0.95} = 16.64$ for $\sigma_G = 0.35$), OPCSR reduces simulated unverified state corruption at $p_{\text{audit}} = 0.10$ and $\rho = 16.0$ to 1.60% [0.81, 3.13] (0.00% [0.00, 0.76] for fixed gains at $\rho \geq 10$) while cutting measured Python verification time by 4.28× (5.97 μs vs. 25.53 μs for full synchronous replay).

Use of AI tools

Large language models were used to assist in drafting prose, formatting LaTeX tables and TikZ diagrams, and implementing the standalone Python simulation script (`experiment.py`). Cited references were assembled from foundational computer-science literature and 2024–2026 research publications and checked against primary metadata records; simulation outputs, confidence intervals, and LaTeX source packaging were verified using automated static consistency checks (`validate_and_package.py`).

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